A rotation of a body in a 3D space in general is expressed by a three dimensional function of time. There exist more than one way to describe a rotation. The most commonly used way is to represent a rotation with three Euler angles.

Roughly speaking, the Earth's rotation can be considered as consisting of two components, the tidally driven component with precisely known frequencies and the component driven by an exchange of the angular momentum between the solid Earth and geophysical fluids. The latter component is not predictable in principle. The atmosphere contributes to the UT1 at a level of 10^{-6} rad, more than three orders of magnitude higher than the accuracy of observations. The first component is also affected by the atmosphere and can be predicted only at a level of 10^{-9} rad. Therefore, the Earth's rotation has to be continuously measured with modern space geodesy techniques.

The Earth rotation is mathematically expressed as a transformation of a vector in the rotating terrestrial coordinate system $\mathbf{r_t}$ to the inertial celestial coordinate $\mathbf{r_c}$. This can be expressed as a product of the rotation matrix with a vector

$$\mathbf{r_c} = \hat{M}_3(E_3(t)) \cdot \hat{M}_2(E_2(t)) \cdot \hat{M}_1(E_1(t)) \quad \mathbf{r_t},$$

where, $E_1(t)$, $E_2(t)$, $E_3(t)$ are Euler angles with respect to axes 1,2,3 and $\hat{M}_X(E_X)$ is a rotation matrix with respect to axis x:

$$M1(E1) = \begin{matrix} 1 & 0 & 0 \\ 0 & \cos E1 & \sin E1 \\ 0 & -\sin E1 & \cos E1 \end{matrix}$$

$$M2(E2) = \begin{matrix} \cos E2 & 0 & -\sin E2 \\ 0 & 1 & 0 \end{matrix}$$

sin E2 0 cos E2

cos E3 sin E3 0

M3(E3) = -sin E3 0 cos E3

0 1 0

However, accordint to the adopted so-called Newcomb-Andoyer formalism, the rotation matrix is decomposed as a product of 12 elementary rotations:

$\hat{M}(t) = \hat{M}_3(\zeta(t)) \cdot \hat{M}_2(-\theta(t)) \cdot \hat{M}_3(z(t)) \cdot \hat{M}_1(-\varepsilon_0(t)) \cdot \hat{M}_3(\Delta\psi(t)) \cdot \hat{M}_1(\varepsilon_0(t) + \Delta\varepsilon(t)) \cdot \hat{M}_3(-S_1(t) + E_3(t)) \cdot \hat{M}_2(E_2(t)) \cdot \hat{M}_1(E_1(t)) \cdot \hat{M}_3(H_3(t)) \cdot \hat{M}_2(H_2(t)) \cdot \hat{M}_1(H_1(t))$

where

- $\zeta(t)$ — the first angle of the precession in right ascension. It is expressed as a lower degree polynomial with respect to TDB argument. TDB (Time Dynamic Barycentric) is a function of TAI.
- $\theta(t)$ — precession declination ascension. It is expressed as a lower degree polynomial with respect to TDB argument.
- $z(t)$ — the second argument of precession in right ascension. It is expressed as a lower degree polynomial with respect to TDB argument.
- $\varepsilon_0(t)$ — the mean inclination of the ecliptic to the equator. It is expressed as a lower degree polynomial with respect to TDB argument.
- $\Delta\psi(t)$ — nutation in longitude. It is expressed in a quasi-harmonic expansion

Σ (a_s(i) + b_s(i) t)*sin (p(i) + q(i) t + 1/2 r(i) t^2) + (a_c(i) + b_c(i) t)*cos (p(i) + q(i) t + 1/2 r(i) t^2) + Ψ_0 + $\dot{\Psi}$ t
where t is the TDB argument.

- $\Delta\varepsilon$(t) — nutation in obliquity. It is expressed in a quasi-harmonic expansion
  Σ (a_c(i) + b_c(i) t)*cos (p(i) + q(i) t + 1/2 r(i) t^2) + (a_s(i) + b_s(i) t)*sin (p(i) + q(i) t + 1/2 r(i) t^2) + E_0 + $\dot{E}$ t
  where t is the TDB argument.

- S_1(t) — modified stellar argument. It is a function of low degree polynomials, ζ(t), θ(t), z(t); ε_0(t), $\Delta\psi$(t), $\Delta\varepsilon$(t)

- E_3(t) — Euler angle around axis 3 (i.e. axial rotation). It is related to a commonly used argument UT1(t) or UT1MTAI(t) (UT1 minus Tai): E_3(t) = - κ UT1(t) = - κ (t - UT1MTAI(t)), where κ = 1.00273781191135448*2π/86400.0. Units for E_3(t), units for UT1(t) or UT1MTAI(t) is seconds. E_3(t) is a slowly variating function of time and is determined from observations.

- E_2(t) — Euler angle around axis 2 (i.e. axial rotation). It is related to a commonly used argument X pole coordinate. E_2(t) is a slowly variating function of time and is determined from observations.

- E_1(t) — Euler angle around axis 1 (i.e. axial rotation). It is related to a commonly used argument Y pole coordinate. E_1(t) is a slowly variating function of time and is determined from observations.

- H_3(t) — Euler angle around axis 3 that holds harmonic variations. It is expressed in a form of quasi-harmonic expansion
  H_3(t) = Σ a_c(i) * cos (p(i) + q(i) t + 1/2 r(i) t^2) + a_s(i) * sin (p(i) + q(i) t + 1/2 r(i) t^2) +

Coefficients of the expansion are determined from analysis of space geodesy observations.

- $H_2(t)$ — Euler angle around axis 2 that holds harmonic variations. It is expressed in a form of quasi-harmonic expansion
  $H_2(t) = \Sigma (a_c(i) + \dot{a}_c) * \sin (p(i) + q(i) t + 1/2 r(i) t^2) - (a_s(i) + \dot{a}_s) * \cos (p(i) + q(i) t + 1/2 r(i) t^2)$
  Coefficients of the expansion are determined from analysis of space geodesy observations.

- $H_1(t)$ — Euler angle around axis 1 that holds harmonic variations. It is expressed in a form of quasi-harmonic expansion
  $H_1(t) = \Sigma (a_c(i) + \dot{a}_c) * \cos (p(i) + q(i) t + 1/2 r(i) t^2) + (a_s(i) + \dot{a}_s) * \sin (p(i) + q(i) t + 1/2 r(i) t^2)$
  Coefficients of the expansion are determined from analysis of space geodesy observations.

The choice of Earth rotation parameterization is not logical, not economical, not optimal. This choice follows a historical tradition. Decomposition of a product of three matrix into a product of 12 matrix can be done by more than one way. There is an alternative decomposition Ginot-Capitaine. That decomposition is entirely equivalent to the Newcomb-Andoyer formalism.

NERS library keeps numerical coefficients of expansion $\zeta(t)$, $\theta(t)$, $z(t)$; $\varepsilon_0(t)$, $\Delta\psi(t)$, $\Delta\varepsilon(t)$, and $S_1(t)$ and has the code that computes them on the specified moment of time. Empirical functions E(t) and H(t) are taken from the *NERS* server EOP message. Function E(t) comes as a table of values on specified, in general non-equidistant epochs. The tables are updated several times a day. Function H(t) comes in a form of a table of expansion coefficients determined from analysis of observations. It is updated 4–6 times a year. See NERS how for explanation how *NERS server generates the EOP message.*

NERS client automatically downloads the EOP message and extracts from there E(t) and H(t) functions relevant to the request, and computes the Earth's *rotation matrix.*

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